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**Abstract**

This paper estimates a sticky price macro model with US macro and term structure data using Bayesian methods. The model is solved by a nonlinear method. The posterior distribution of the parameters in the model is found to be bi-modal. The degree of nominal rigidity is high at one mode ("sticky price mode") but is low at the other mode ("flexible price mode"). I find that the degree of nominal rigidity is important for identifying macro shocks that affect the yield curve. When prices are more flexible, a slowly varying inflation target of the central bank is the main driver of the overall level of the yield curve by changing long-run inflation expectations. In contrast, when prices are more sticky, a highly persistent markup shock is the main driver. The posterior probability of each mode is sensitive to the use of observed proxies for inflation expectations. Ignoring additional information from survey data on inflation expectations significantly reduces the posterior probability of the flexible price mode. Incorporating this additional information suggests that yield curve fluctuations can be better understood by focusing on the flexible price mode. Considering nonlinearities of the model solution also increases the posterior probability of the flexible price mode, although to a lesser degree than using survey data information.

***Keywords:*** Bayesian Econometrics, DSGE Model, Term Structure of Interest Rates

***JEL Classification:*** C32, E43, G12

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[*] Economic Research Department, Federal Reserve Bank of Kansas City, One Memorial Drive, Kansas City, MO, USA 64198; email: taeyoung.doh@kc.frb.org The appendix for technical results is available online at www.taeyoung-doh.net. The views expressed herein are solely those of the author and do not necessarily reflect the views of the Federal Reserve Bank of Kansas City or the Federal Reserve System.

# 1 Introduction

Dynamic term structure models that use a few factors to explain changes in the shape of the entire yield curve are empirically successful.[1] In these models, factors are typically extracted from a statistical decomposition of the yield curve. However, the economic interpretation of such statistical factors is not clear. Recent empirical studies on the macroeconomics of the term structure (e.g., Ang et al., 2003, Bikbov et al., 2010, and Diebold et al., 2006) show a close link between macroeconomic variables and bond prices. These studies augment statistical factors of the yield curve with macroeconomic variables. Despite the inclusion of macro variables, latent term structure factors without a clear economic interpretation still explain a significant portion of the variation of the yield curve.

In this paper, I set up and estimate a New Keynesian dynamic stochastic general equilibrium (DSGE) model to explain the joint fluctuations of macroeconomic variables and the yield curve. In the model, four different shocks drive economic fluctuations. They are shocks to technology, firms' price markups, the inflation target of the central bank, and a transitory monetary policy shock. I do not add latent term structure factors that are orthogonal to macro shocks and instead try to maximize the explanatory power of macro factors. By linking the estimates of shocks with empirical counterparts of latent term structure factors, I provide an economic interpretation of these purely statistical factors. In addition, the DSGE framework can shed light on the kind of endogenous amplification channels that can account for how these macro shocks drive yield curve fluctuations. Such an explanation is not possible to explore in factor models of the yield curve augmented with observed macro variables.

This paper uses a second-order approximate solution in the estimation of the DSGE model. There are two reasons for this approach. First, Fernàndez-Villaverde et al. (2006), An (2005), and Amisano and Tristani (2007) show that there are noticeable differences in the likelihood and parameter estimates across first-order and second-order solutions. These differences are large when data are highly persistent. Bond yields have this property (See Figure 1). Therefore, one can expect nonlinearities to be important in the estimation with yield curve data. Second, the first-order accurate solution of a DSGE model ignores terms which can contribute to term

[1] For the assessments of fit of empirical factor models of the yield curve, see Singleton (2006, Chap. 13)

premia. I propose a method to analytically evaluate conditional expectations of no-arbitrage conditions for bond yields, based on the stochastic discount factor given by a second-order solution of the DSGE model. This approach differs from Hördahl et al. (2008), Ravenna and Seppälä (2006), and Rudebusch and Swanson (2008) who use various approximations for bond yields on top of a higher-order approximation to the macro solution.

Three main findings are obtained from this study. First, the posterior distribution of the parameters in the model is found to be bi-modal. Posterior probability is much higher for the mode with a high degree of nominal rigidity ("sticky price mode") than the mode with a low degree of nominal rigidity ("flexible price mode"). However, the posterior probability of each mode is sensitive to the inclusion of observed proxies for inflation expectations from the survey of professional forecasters. Since the flexible price mode captures the time-variation of survey data better than the sticky price mode, including this additional information substantially increases the posterior probability of the flexible price mode.

Second, nominal rigidity is important in identifying the macro factors of the yield curve. When prices are more flexible, the low frequency movements of inflation and the overall level of the yield curve are mostly driven by nominal disturbances. But if prices are sticky, real disturbances matter more. The degree of nominal rigidity also determines which shocks account for the slope of the yield curve. For instance, when nominal rigidity is low, markup shocks are the main drivers of the slope; whereas, when nominal rigidity is high, monetary policy shocks dominate.

Third, the nonlinearities of the model solution are also important for assessing the posterior probability of each mode. Ignoring nonlinearities of inflation dynamics reduces the posterior probability of the flexible price mode, although to a lesser degree than using survey data information.

This paper is related to the literature linking estimated macro shocks obtained from DSGE models with the yield curve. Evans and Marshall (2007) use empirical measures of macro shocks to identify economic determinants of the nominal treasury bond yields. They argue that the systematic component of monetary policy is important in linking macro shocks with the yield curve. This paper also finds the importance of the systematic response of the policy rate in describing the way macro shocks influence the yield curve. However, the way I identify macro shocks is different. In Evans and Marshall (2007), some shocks are obtained from using first-order conditions of a DSGE model at the calibrated parameter values, whereas other shocks are obtained from using identifying restrictions in structural vector autoregressions from other papers. Therefore, the internal consistency of these measures is not clear.[2] In contrast, this paper imposes restrictions of a single DSGE model to identify all the macro shocks. Another closely related paper is Bekaert et al. (2006) who combine the log-linear solution of a stylized New Keynesian model with the log-normality of the approximate pricing kernel. Their interpretation that the time-varying inflation target of the central bank is the main factor that explains the parallel shifts in the yield curve, is in line with this paper. However, their use of the log-linear solution of the macro model ignores the role of nonlinear terms in the model solution. More importantly, neither of these studies discusses the role of nominal rigidity in identifying macro shocks driving the yield curve, which is the main focus of this paper.

Additional literature closely related to this paper explores term structure implications of New Keynesian DSGE models solved with nonlinear methods (Hördahl et al., 2008, Ravenna et al., 2006, and Rudebusch et al., 2008 etc.).[3] After reviewing the results of various papers, Rudebusch and Swanson (2008) conclude that stylized New Keynesian models have a hard time matching the first and second moments of term premia without compromising macro implications. In line with this finding, the model-implied term premia of the DSGE model studied in this paper are too stable compared to what is obtained by a reduced-form benchmark model.

The remainder of the paper is organized as follows. Section 2 describes the model economy and presents a second-order approximation to model's solution and proposes a new method to derive equilibrium bond yields based on the second-order approximation. Section 3 describes

[2] A similar issue exists in Rudebusch and Wu (2008) who use a log-linearized New Keynesian model to describe macro dynamics but derive equilibrium bond yields from an arbitrary pricing kernel inconsistent with the pricing kernel given by the New Keynesian model.

[3] In terms of estimating a nonlinear solution of the DSGE model, this paper is related to Fernàndez-Villaverde et al. (2006), An (2005), Amisano and Tristani (2007), and Binsbergen et. al. (2010). In particular, Binsbergen et. al. (2010) include the yield curve data in the estimation as this paper does while other papers use only macro data.

data and the econometric methodology. Sections 4 and 5 discuss the estimation results, and Section 6 concludes.

## 2 Model Economy

This paper uses a small-scale New Keynesian model similar to the one discussed in Woodford (2003). Below I will provide a brief description of the model, which closely follows An and Schorfheide (2007).

### 2.1 Private Agents

Perfectly competitive firms produce the final consumption good $Y_t$ using the intermediate goods $Y_t(j)$ , $j \in [0, 1]$ and the production technology

$$Y_t = \left( \int_0^1 Y_t(j)^{\frac{\zeta_t - 1}{\zeta_t}} dj \right)^{\frac{\zeta_t}{\zeta_t - 1}}, \tag{1}$$

where $\zeta_t > 1$ represents the elasticity of demand for intermediate goods.

Profit maximization and zero profit condition for the final goods producers imply the following demand function for the intermediate good $j$:

$$Y_t(j) = (\frac{P_t(j)}{P_t})^{-\zeta_t} Y_t. \tag{2}$$

Here, $P_t$ is the price of the final good and $P_t(j)$ is the price of the intermediate good $j$.

Production technology for intermediate good $j$ is linear with respect to labor and the relationship is given by,

$$Y_t(j) = A_t N_t(j), \tag{3}$$

where $A_t$ is a technology shock common to all the firms and $N_t(j)$ is the labor input of firm $j$.

Firms in the intermediate goods sector face nominal rigidities in the form of quadratic price adjustment costs,

$$AC_t(j) = \frac{\phi}{2}(\frac{P_t(j)}{P_{t-1}(j)} - \pi^\star)^2 Y_t(j). \tag{4}$$

where $\phi$ is a parameter governing the degree of nominal rigidity in this economy and $\pi^\star$ is the steady state inflation rate in terms of the final good.

The labor market is assumed to be perfectly competitive and the real wage is denoted by $W_t$. Firm $j$ decides its labor input $N_t(j)$ and the price $P_t(j)$ that maximize the present value of its profit stream[4]

$$\mathrm{E}_t[\sum_{s=0}^{\infty}\beta^s\frac{\lambda_{t+s}}{\lambda_t}(\frac{P_{t+s}(j)}{P_{t+s}}Y_{t+s}(j)-W_{t+s}N_{t+s}(j)-AC_{t+s}(j))], \tag{5}$$

where $\lambda_{t+s}$ is the marginal utility of a final good to the representative household at time $t+s$.

The representative household maximizes its utility by choosing consumption ($C_t$) and labor supply($H_t$).[5] I deflate consumption by the current technology level to ensure a balanced growth path for the economy. Also, I introduce a form of internal habit formation into the utility function.[6] The representative household wants to maximize her expected discounted lifetime utility,

$$\mathrm{E}_t[\sum_{s=0}^{\infty}\beta^s(\frac{(C^a_{t+s}/A_{t+s})^{1-\tau}-1}{1-\tau}-\frac{H_{t+s}^{1+\frac{1}{\nu}}}{1+\frac{1}{\nu}})], \tag{6}$$

where $C^a_{t+s}=C_{t+s}-he^{u_a^\star}C_{t+s-1}$ is consumption relative to the habit level, which is determined by the previous period consumption, $h$ is the parameter governing the magnitude of habit persistence, $u_a^\star$ is the steady state rate of technology progress, $\tau$ is the curvature of utility function, and $\nu$ is the short-run (Frisch) labor supply elasticity. Under the assumption of complete asset markets, the household's budget constraint is of the form

$$P_tC_t+\sum_{n=1}^{\infty}P_{n,t}(B_{n,t}-B_{n+1,t-1})+T_t=P_tW_tH_t+B_{1,t-1}+Q_t+\Pi_t, \tag{7}$$

where $P_{n,t}$ is the price of an $n$ quarter bond, $B_{n,t}$ is the holding of the $n$ quarter bond, $T_t$ is the lump-sum tax or subsidy, $Q_t$ is the net cash flow from participating in state-contingent security markets, and $\Pi_t$ is the aggregate profit.

[4]The price of the final good is given by $P_t=\left(\int_0^1 P_t(j)^{1-\zeta_t}dj\right)^{\frac{1}{1-\zeta_t}}$.

[5]I do not model money explicitly. If I introduce real money balances which are separable from other arguments in the utility function, all the following arguments go through.

[6]This preference specification corresponds to the closed economy version of Lubik and Schorfheide (2005). As Hördahl et al. (2008) illustrate, habit formation is important to jointly explain consumption and the yield curve.

### 2.2 Monetary and Fiscal Policies

The monetary policy of the central bank follows a forward-looking Taylor rule with interest rate smoothing. The nominal interest rate reacts to expected inflation and output gap in the following way,

$$\begin{aligned}(1+i_t) &= (1+i_t^\star)^{1-\rho_i}(1+i_{t-1})^{\rho_i}e^{\eta_i\epsilon_{i,t}},\ \epsilon_{i,t}\ i.i.d.\mathcal{N}(0,1)\\ 1+i_t^\star &= ((1+r^\star)(\pi^\star))(\tfrac{E_t(\pi_{t+1})}{\pi_t^\star})^{\gamma_p}(\tfrac{Y_t}{A_t y^\star})^{\gamma_y},\end{aligned} \tag{8}$$

where $r^\star$ is the steady state real interest rate, equal to $\frac{e^{u_a^\star}}{\beta}-1$, $\pi_t$ is the gross inflation rate defined by $\frac{P_t}{P_{t-1}}$, $\pi_t^\star$ is the time varying inflation target of the central bank, $y^\star$ is the steady state value of the detrended output $\frac{Y_t}{A_t}$, and $\epsilon_{i,t}$ is a monetary policy shock. In the model, the time varying inflation target is assumed to be exogenously given and $\rho_i$ captures the degree of interest rate smoothing.

The fiscal side of government policy is passive. The fiscal authority collects money from levying lump-sum taxes and issuing new bonds. It also provides lump-sum subsidies and repays maturing bonds. The government is subject to the following period-by-period budget constraint, $\sum_{n=1}^{\infty} P_{n,t}(B_{n,t}-B_{n+1,t-1})+T_t = B_{1,t-1}$.

### 2.3 Exogenous Processes

There are four structural disturbances in the model. The first two are real disturbances that affect aggregate technology and price markups. Aggregate productivity evolves according to,

$$u_{a,t} = \frac{A_t}{A_{t-1}}\ ,\ \ln u_{a,t+1} = (1-\rho_a)u_a^\star + \rho_a \ln u_{a,t} + \eta_a\epsilon_{a,t+1}, \tag{9}$$

where $\epsilon_{a,t+1}$ is $i.i.d.\mathcal{N}(0,1)$.

If there were no nominal rigidities, firms in the intermediate goods sector would set their price-marginal cost ratios equal to $\frac{\zeta_t}{\zeta_t-1}$. These ratios are called the desired markups of firms. In the model, these ratios are time varying and follow in logs the AR(1) process:

$$f_t = \frac{\zeta_t}{\zeta_t-1}\ ,\ \ln f_{t+1} = (1-\rho_f)\ln f^\star + \rho_f \ln f_t + \eta_f\epsilon_{f,t+1}, \tag{10}$$

where $\epsilon_{f,t+1}$ is $i.i.d.\mathcal{N}(0,1)$.

Two nominal disturbances are a serially uncorrelated monetary policy shock ($\epsilon_{i,t}$) and a persistent shock to the inflation target of the central bank ($\ln \pi^\star_t$) that follows an AR (1) process,

$$\ln \pi^\star_{t+1} = (1-\rho_{\pi^\star}) \ln \pi^\star + \rho_{\pi^\star} \ln \pi^\star_t + \eta_{\pi^\star} \epsilon_{\pi^\star,t+1}, \tag{11}$$

where $\epsilon_{\pi^\star,t+1}$ is $i.i.d.\mathcal{N}(0,1)$.

### 2.4 Equilibrium Conditions

Market clearing conditions for the final good market and labor market are given by

$$Y_t = C_t + AC_t \ , \ H_t = N_t. \tag{12}$$

The first-order conditions for firms and the representative household can be expressed as follows:

$$\begin{aligned}
\lambda_t A_t &= \lambda^a_t = (C^a_t/A_t)^{-\tau} - \beta h e^{u^\star_a} E_t((C^a_{t+1}/A_{t+1})^{-\tau} A_t/A_{t+1}), \\
1 &= \beta \mathrm{E}_t[(\frac{\lambda^a_{t+1}}{\lambda^a_t}) \frac{A_t}{A_{t+1}} \frac{1+i_t}{\pi_{t+1}}], \qquad (13)\\
1 &= \zeta_t[1 - \frac{(\frac{Y_t}{A_t})^{\frac{1}{\nu}}}{\lambda^a_t}] + \phi\pi_t(\pi_t - \pi^\star) - \frac{\phi}{2}\zeta_t(\pi_t - \pi^\star)^2 \\
&- \phi\beta \mathrm{E}_t[(\frac{\lambda^a_{t+1}}{\lambda^a_t}) \frac{Y_{t+1}/A_{t+1}}{Y_t/A_t} \pi_{t+1}(\pi_{t+1} - \pi^\star)]. \qquad (14)
\end{aligned}$$

### 2.5 Model Solution

The model is solved using a perturbation method for equilibrium conditions around the non-stochastic steady state. Since the non-stationary trend of technology induces a stochastic trend in output and consumption, it is convenient to express the model in terms of detrended variables $c_t = C_t/A_t$ and $y_t = Y_t/A_t$. For notational convenience, the percentage deviation of a variable $d_t$ from its steady state $d^\star$ is denoted by $\hat{d}_t = \ln(d_t/d^\star)$.

Equations (8) - (14), which consist of equilibrium conditions and specifications for exogenous processes, form the following rational expectations system of equations:

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t, \sigma\epsilon_{t+1}) = 0 \tag{15}$$

$$\begin{aligned}
&\sigma \in [0,1] \ , \ \epsilon_{t+1} = [\epsilon_{a,t+1}, \epsilon_{f,t+1}, \epsilon_{i,t+1}, \epsilon_{\pi^\star,t+1}] \\
&y_t = [\widehat{Y_t/A_t}, \widehat{C_t/A_t}, \hat{\pi}_t, \widehat{(1+i_t)}, \widehat{C^a_t/A_t}, \hat{\lambda}^a_t] \quad : \quad \text{(control variable)}, \\
&x_t = [\hat{u}_{a,t}, \hat{f}_t, \epsilon_{i,t}, \hat{\pi}^\star_t, \widehat{(1+i_{t-1})}, \widehat{C_{t-1}/A_{t-1}}] \quad : \quad \text{(state variable)}.
\end{aligned}$$

$\sigma$ is a perturbation parameter that determines the distance from the deterministic steady state. Hence, $\sigma = 0$ corresponds to the non-stochastic steady state. The approximate economy is associated with $\sigma = 1$. Since $\sigma\epsilon_{t+1}$ is the only source of uncertainty, the approximation order in the perturbed system is determined by the degree of powers of $\sigma$ in the approximated system.

Following Schmitt-Grohé and Uribe (2004), I obtain the second-order approximate solution of (15). The resulting approximate solution is as follows:[7]

$$\begin{aligned} y_t &\approx \frac{1}{2}g_{\sigma\sigma}\sigma^2 + g_x x_t + \frac{1}{2}(I_{n_y} \otimes x_t)'(g_{xx})x_t, && (16) \\ x_{t+1} &\approx \frac{1}{2}h_{\sigma\sigma}\sigma^2 + h_x x_t + \frac{1}{2}(I_{n_x} \otimes x_t)'(h_{xx})x_t + \sigma\eta\epsilon_{t+1}. && (17) \end{aligned}$$

Equation (16) describes how control variables respond to current state variables and equation (17) provides transition equations to the state variables.

### 2.6 Equilibrium bond yields

In the model, the one-period ahead nominal stochastic discount factor ($M_{t,t+1}$) is given by,

$$M_{t,t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}\frac{1}{\pi_{t+1}}. \tag{18}$$

The approximate solution also provides the following approximation to the log stochastic discount factor of the model,

$$\hat{M}_{t,t+1} = (\hat{\lambda}^a_{t+1} - \hat{\lambda}^a_t) - \hat{u}_{a,t+1} - \hat{\pi}_{t+1} = m_0 + m_1 x_t + x'_t m_2 x_t + m_3 x_{t+1} + x'_{t+1} m_4 x_{t+1}. \tag{19}$$

The one-period holding return of an $n$ quarter bond is $e^{\hat{p}_{n-1,t+1}-\hat{p}_{n,t}}$. The absence of arbitrage opportunities implies that the gross expected return of any bond should be equal to one if we adjust the risk of holding the bond for one period, based on the approximate stochastic discount factor.

$$1 = E_t(e^{\hat{M}_{t,t+1}+\hat{p}_{n-1,t+1}-\hat{p}_{n,t}})\ ,\ \hat{p}_{1,t} = -\widehat{1+i_t}. \tag{20}$$

The challenge in solving for bond prices is to compute conditional expectations in the above equation when the exact distribution of $\hat{M}_{t,t+1}$ is complicated due to quadratic terms. In the

[7]The representation of solutions follows Klein (2005).

linear model, $\hat{M}_{t,t+1}$ is log-normally distributed and conditional expectations can be easily evaluated, as in Jermann (1998); however, the approximate stochastic discount factor in equation (18) is not log-normally distributed. Instead of trying to characterize the distribution of $\hat{M}_{t,t+1}$, I derive log bond prices as quadratic functions of state variables using the distribution of $\epsilon_{t+1}$.[8]

$$\hat{p}_{n,t} = a_n + b_n x_t + x_t' c_n x_t.$$

The yield of an $n$-quarter bond $\hat{y}_{n,t}$ can be computed as $-\frac{\hat{p}_{n,t}}{n}$. Therefore, equilibrium bond yields are given by,

$$y_{n,t} = \ln(1+i^\star) - \frac{a_n}{n} - \frac{b_n}{n} x_t - x_t' \frac{c_n}{n} x_t. \tag{21}$$

## 3 Data and Econometric Methodology

The DSGE model is estimated using Bayesian methods. This section describes data and the econometric methodology.

### 3.1 Data

The dataset consists of US macro and treasury bond yields. Macro variables are taken from the Federal Reserve Economic Data (FRED) available on the Federal Reserve Bank of Saint Louis website. The measure of output is per-capita real GDP, obtained by dividing real GDP (GDPC1) by total population (POP). The inflation rate is the log difference of the GDP deflator (GDPCTPI). The nominal interest rate is the 3-month treasury bill rate from the Fama CRSP risk-free rate file. Bond yields (1,2,3,4, and 5 year) are obtained from Fama-Bliss

[8] John and Kotz (1970) provide formulas for expectations of exponential quadratic forms of multivariate normal random variables. Ang, Boivin, and Dong (2007) use a similar method to derive bond prices in a model with drifting policy parameters in a Taylor rule. The DSGE setup in this paper introduces additional complications since some state variables follow a quadratic law of motion. Mechanical applications of the second-order approximation for $x_{t+1}^2$ generate extra higher-order terms such as $x_t^4$ and $x_t^3$. As discussed in Kim et.al. (2008), these extra higher-order terms do not necessarily increase the accuracy of the approximation. Following their suggestion, I prune these extra higher-order terms and consider terms up to the second-order of $(x_t, \epsilon_{t+1})$. The combination of the pruning scheme with the exponential quadratic forms of multi-variate normal random variables results in equilibrium bond yields as quadratic functions of state variables in this case. The details of the derivation are explained in the web appendix (available on www.taeyoung-doh.net).

CRSP discount bond yields files. The sample period is from 1983:QI to 2007:QIV. To match the frequency of the yields with that of macro data, observations of the treasury bill rate and bond yields are transformed into quarterly data by arithmetic averaging. The sample data are plotted in Figure 1.

### 3.2 Econometric methodology

The approximate solution and the derivation of equilibrium bond yields provide the law of motion for state variables and the measurement equations for observed macro variables and bond yields. Put together, these equations result in the following nonlinear state space model:

$$
\begin{aligned}
x_t &= \Gamma_0(\vartheta) + \Gamma_1(\vartheta)x_{t-1} + (I_{n_x} \otimes x_{t-1})'\Gamma_2(\vartheta)x_{t-1} + \sigma\eta\epsilon_t, && (22)\\
z_t &= \alpha_0(\vartheta) + \alpha_1(\vartheta)x_t + (I_{n_z} \otimes x_t)'\alpha_2(\vartheta)x_t + \xi_t \text{ where } \xi_t \sim \mathcal{N}(0, H), && (23)\\
z_t &= [\ln Y_t, \ln(1+\pi_t), \ln(1+i_t), y_{4,t}, y_{8,t}, y_{12,t}, y_{16,t}, y_{20,t}]',\\
\vartheta &= [\tau, \beta, \nu, \ln f^\star, \phi, u_a^\star, \gamma_p, \gamma_y, \rho_a, \rho_f, \rho_i, \rho_{\pi^\star}, \eta_a, \eta_f, \eta_i, \eta_{\pi^\star}, \ln A_0, \ln \pi_0^\star, h]',
\end{aligned}
$$

where $z_t$ and $\xi_t$ are a set of observed variables and a vector of measurement errors, respectively. $H$ is assumed to be a diagonal matrix, which implies that the measurement errors are independent across observed variables. Standard deviations of measurement errors are calibrated due to complications from estimating these parameters in nonlinear models.[9]

To evaluate the likelihood, I integrate out unobserved state variables based on the filtering density $p(x_t|z^t, \vartheta)$.[10] This density does not have an analytical form because of the nonlinearities in the state-space representation. I use simulation-based particle filtering to approximate this density by a large swarm of particles $x_t^i$ $(i = 1, \cdots N)$, as in An (2005) and Fernández-Villaverde and Rubio-Ramírez (2007).

This paper uses Bayesian methods that combine prior information on parameters with the likelihood. One advantage of incorporating prior information is that it places less emphasis on regions of the parameter space that are at odds with observations not included in the estimation sample. This property is particularly relevant for the estimation of DSGE models

[9]This is mainly because the standard deviations of measurement errors play the same role as bandwidths in nonparametric estimation. The analogy is explained in the web appendix.

[10]Here, $z^t$ denotes observations up to time $t$ , $[z_1, \cdots, z_t]$.

because the likelihood function often peaks in regions of the parameter space that appear to be inconsistent with extraneous information that researchers have.[11] The prior information on structural parameters is represented by the prior density $p(\vartheta)$. All the parameters are assumed to be independent *a priori*.[12] The posterior density is proportional to the posterior kernel which is the product of the prior density and likelihood,

$$p(\vartheta|z^T) \propto p(\vartheta)\mathcal{L}(\vartheta|z^T). \tag{24}$$

The analytical form of the posterior density is not known, but I can generate parameter draws whose distributions converge to the posterior distributions by using Markov Chain Monte Carlo (MCMC) methods, as explained in An and Schorfheide (2007).

# 4 Estimation Results

This section discusses parameter estimates and the model fit using posterior draws of parameters and smoothed estimates of measurement errors.

## 4.1 Prior distribution

The specification for the prior distribution is summarized in Table 1. I set prior means of parameters determining the steady state of the model by matching implied steady state values with the average observations of the pre-sample period. This is similar to the calibration exercise in Cooley and Prescott (1995). Reflecting information contained in the pre-sample data, I impose tight prior standard deviations for these parameters. A few parameters are fixed at prior means for technical reasons. For example, because hours worked are not used in the estimation, the elasticity of labor supply will not be well identified with sample information. This parameter is fixed at 0.5, close to the posterior mean estimate in Chang et. al. (2007).[13]

[11] An and Schorfheide (2007) provide more detailed discussions on this issue.

[12] While I draw parameters independently from prior distributions, I eliminate parameters implying the indeterminacy of equilibrium in the linear model. As a result, the distribution of the remaining parameter draws may not be independent.

[13] Also, standard deviations of measurement errors of output, inflation rate, and bond yields are fixed at about 20% of the sample standard deviations of output growth, inflation rate, and the nominal interest rate. The web

For other parameters, I set relatively loose priors to enhance the model's ex-ante explanatory power. One exception to this rule is the parameter characterizing monetary policy response to inflation. When this parameter is below one, many parameter draws imply the indeterminacy of equilibrium in the linear model. Since I throw away parameter draws implying the indeterminacy, I set a relatively tight prior for this parameter to reduce chances of hitting the indeterminacy region.

## 4.2 Posterior distribution

By running multiple MCMC chains from different starting points, I find that the posterior density is high around the two local modes reported in Table 2 but the area between the two modes has a very low posterior density. At one mode, the price adjustment cost parameter is low and this mode can be called the "flexible price mode". At another mode, the price adjustment cost parameter is high and this mode can be called the "sticky price mode".[14]

While results from multiple MCMC chains reveal the bi-modality of the posterior distribution of parameters, they do not provide information on the posterior masses of areas around the two modes because a single MCMC chain does not visit both areas at the same time. However, correctly evaluating posterior masses is important because two modes provide starkly different explanation on the sources of persistence in inflation. When prices are more flexible, nominal disturbances affect prices more than real output. When prices are sticky, nominal disturbances can have greater impacts on real output. Since macro shocks driving the persistence of inflation can also account for persistent movements of long-term bond yields through long-run inflation expectations, the evaluation of the posterior mass is also important for understanding yield curve fluctuations. For this purpose, I run another MCMC chain that can explore a wider area of parameter space.[15]

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appendix discusses why this calibration is reasonable.

[14]The bi-modality indicates that there might be identification issues for some parameters. These issues can arise either from data-related reasons or model-related reasons according to Canova and Sala (2009). Estimation results of the DSGE model using data simulated from the same model do not show evidence of such a bi-modality. Also, if we use only macro data in the estimation, the bi-modality disappears. Including the highly persistent yield curve data in the estimation seems to complicate the identification of parameters determining the sources of persistence of endogenous variables in the model.

[15]To facilitate the crossing between the two modes, I use a mixture of $t$-distributions as the proposal density

The posterior densities of parameters in Figures 2-3 show that the sticky price area has a much higher posterior mass. Also, the sticky price mode matches the second moments of short rate and near-term maturity bond yields better than the flexible price mode, as shown in Table 3.

Can we ignore the flexible price area in posterior inference based on these results? In fact, if we ignore nonlinearities of the model solution and estimate the log-linearized DSGE model, the flexible price mode has an essentially zero posterior mass. However, there is some empirical evidence that the area should not be ignored. Indeed, I will argue that we should focus on the flexible price mode to jointly explain macro variables and yield curve if we want to incorporate additional information from observed proxies for inflation expectations. Table 4 shows the correlation between the model implied one year ahead inflation expectations and the corresponding variable in the survey data. The correlation is much higher at the flexible price mode than the sticky price mode. This result suggests that the flexible price mode can have a higher posterior probability than the sticky price mode when the survey data are included in the estimation. This turns out to be true when I estimate the linear model with the dataset augmented with survey data. The log-likelihood of each mode reported in Table 5 shows that now the log-likelihood is much higher at the flexible price mode.[16] Since the flexible price mode is more consistent with a broad set of data, I will focus on term structure implications of the DSGE model at the flexible price mode.

To a lesser degree than using information outside of the model, considering nonlinearities in the model solution is also important in evaluating the plausibility of each mode. While quadratic terms in the law of motion for inflation do not matter much quantitatively at the sticky price mode, they have significant impacts at the flexible price mode. This feature is crucial for simultaneously fitting volatile long-term bond yields and less volatile inflation. The DSGE model at the flexible price mode generates volatile long-run inflation expectations through highly persistent shocks to the inflation target of the central bank. Transitory mon-

in which the local mode of each area is captured in components of the proposal density.

[16]The details of estimation results are available on the web appendix. Estimating a nonlinear model with survey data is computationally very challenging because the model should generate inflation expectations for each particle at each period.

etary policy shocks can dampen the volatility of near-term inflation if they raise policy rate when inflation target is high. This dampening effect of transitory monetary policy shocks is much bigger when we incorporate quadratic terms in the law of motion for inflation. Hence, ignoring nonlinearities substantially reduces the likelihood at the flexible price mode by generating too volatile inflation.[17] The finding suggests a potential pitfall in ignoring nonlinearities of the DSGE model.

### 4.3 Model fit

The model's ability to fit the data can be assessed by looking at smoothed estimates of measurement errors. I extract smoothed estimates of measurement errors $E(\xi_t|z^T, \vartheta)$ and compute their means and standard deviations. The absolute values of the means of ex-post measurement errors of nominal variables in Table 6 are close to zero, with small standard deviations of estimated measurement errors. During the estimation, standard deviations of the measurement errors of bond yields were fixed at roughly 20% of standard deviations of bond yields. For the estimated measurements errors, the corresponding values lie between 6% and 10%. Smaller values of standard deviations of ex-post measurement errors compared to the calibrated values imply that the calibration did not impose severe constraints for the model fit.

## 5 Economic Determinants of the Yield Curve

Based on the estimation results at the flexible price mode, I discuss the economic determinants of the yield curve in the estimated nonlinear model. First, I discuss how nominal rigidity and shock persistence interact in the model to explain the persistence of the yield curve data. Then, I use smoothed estimates of macro shocks to identify underlying macro factors driving the yield curve. In particular, the estimates of macro shocks are linked with the three statistical yield curve factors widely used in the empirical literature on the yield curve. Finally, I compare the term premium implied by the DSGE model with the one obtained from a reduced-form model.

[17]The web appendix provides further details on this issue

## 5.1 Nominal rigidity and persistence of shocks

The persistence of endogenous variables is determined by both endogenous propagation and the persistence of exogenous shocks. Among parameters that determine the endogenous propagation mechanism, only the risk aversion parameter ($\tau$) and the degree of nominal rigidity ($\phi$) show significant differences across the two local modes of the nonlinear model. However, I focus only on the degree of nominal rigidity because the risk aversion parameter does not change much the persistence properties of endogenous variables. To disentangle the role of nominal rigidity from that of shock persistence in determining the persistence of endogenous variables, I plot dynamic responses of endogenous variables to exogenous shocks at different parameter values in Figure 4.[18]

For the first set of parameter values, I use parameter values from the flexible price mode and impulse-responses are in the blue solid lines. The second set of parameter values are obtained by increasing the value of $\phi$ to the value from the sticky price mode, while keeping all the other parameter values the same as in the first set. Impulse-responses in this case are in the red dashed line. The third set of parameter values correspond to the sticky price mode. As expected, increasing $\phi$ without changing other parameters strengthens the impact of nominal disturbances, such as monetary policy shocks and inflation target shocks, on output, while the impacts of these disturbances on inflation are somewhat mitigated.

More substantial changes in impulse-responses show up when I also change persistence parameters. Responses to a markup shock and an inflation target shock vary widely depending on the relative magnitude of their persistence. At the flexible price mode, the inflation target shock is the most persistent shock and drives the persistence of inflation and bond yields. In contrast, at the sticky price mode, the markup shock is the most persistent shock and moves the persistence of inflation and bond yields. Accordingly, two modes attribute fluctuations in inflation and the five-year bond yield to different shocks as shown in forecast error variance decomposition in Table 7. At long horizon, the impacts of different estimates of shock persistence at two modes are more noticeable than at short horizon. In addition, a more persistent shock

[18] Since the model has nonlinearities, I use generalized impulse responses computed by the difference in conditional expectations depending on the existence of a particular shock. See Koop, Pesaran, and Potter (1996) for details of implementing this method.

explains a much bigger portion of forecast error variance of the five-year bond yield than that of inflation even at short horizon. This finding is consistent with the view that information on long-horizon inflation expectations is embedded in the long-term bond yield.

The quantitative changes in impulse-responses at different parameter values may suggest that the nominal rigidity is not that much important as a propagation mechanism. However, the degree of nominal rigidity matters not only in determining just the magnitude of impacts of nominal disturbances on real variables but also in determining the persistence of these disturbances. In fact, posterior distributions for the degree of nominal rigidity and parameters governing the persistence of exogenous shocks are highly correlated. For example, when we increase the degree of nominal rigidity from the flexible price mode while keeping persistence parameters the same, the likelihood substantially declines. Increased nominal rigidity reduces the impact of inflation target shock on long-term bond yields, as shown in Figures 4. To fit the volatility of long-term bond yields, now the model induces bigger fluctuations in markup shocks. This change leads to very volatile inflation and short rate, and the model fit deteriorates in this dimension.

## 5.2 Macro factors and yield curve factors

In the empirical finance literature, bond yields are typically modelled as functions of a few latent factors. With the specification of the factor dynamics, we can build dynamic term structure models that can be taken to the data. In many models, we obtain these factors by a purely statistical decomposition of the yield curve. The principal components of the yield curve, which are constructed by the eigenvalue-eigenvector representation of the covariance matrix of bond yields, are commonly used as risk factors explaining the entire yield curve.[19] It turns out that the first two or three components explain almost 99% of the variation of the entire yield curve.[20] The first three components are typically called "level", "slope", and "curvature", reflecting their loadings onto different bond yields. Since these factors do not have a clear economic interpretation, they are linked with observed macroeconomic variables (e.g.,

[19]See the discussion in Singleton (2006, Chap. 12).

[20]The early empirical evidence for the explanatory power of a few principal components for the entire yield curve is Litterman and Scheinkman (1991).

Ang et al., 2003, Diebold et al. 2006), empirical measures of economic shocks (e.g.,Evans et al., 2007), or macro shocks in the log-linearized DSGE models (e.g., Bekaert et al., 2010). I link the first three principal components of the yield curve with the estimated macro shocks from the DSGE model as in Bekaert et al. (2010) but use a nonlinear solution of the DSGE model.

Panel A of Table 8 provides results of regressing statistical yield curve factors on three observed macro variables: output growth, inflation, and short-term interest rate. The adjusted $R^2$ for the yield curve level is 0.927 and indicates that incorporating information from the short-term interest rate can be enough to explain the most persistent component of the yield curve factors, since the short-term interest rate itself is highly persistent. In contrast, adjusted $R^2$s for the slope factor and the curvature factor are only 0.113 and 0.038, respectively. This result may imply that macro factors play only a limited role in explaining the slope and the curvature of the yield curve. However, regression results of the same yield curve factors on estimated macro shocks in panel B of Table 8 suggest that macro factors can explain the yield curve slope relatively well. Here, adjusted $R^2$ for the slope factor is now 0.640. The finding suggests that a substantial portion of changes in the yield curve slope can be explained by macro factors.

The time series plots of estimated macro factors and yield curve factors in Figure 5 highlight specific macro factors responsible for explaining the time series of each yield curve factor. The gradually declining inflation target matches the time variation of the yield curve level. What is the mechanism through which the time-varying inflation target drives the yield curve level? Since the inflation target is highly persistent and nominal rigidity is low, its variation induces changes in long-horizon inflation expectations as well as near-term inflation expectations. As a result, the entire yield curve shifts.

On the other hand, fluctuations in the yield curve slope vary negatively with markup shocks. This correlation implies that the yield curve flattens in response to a positive markup shock. Responses of inflation and output to a positive markup shock in Figures 4 at the flexible price mode explain why the yield curve flattens. Since a markup shock is moderately persistent and price adjustment costs are small, inflation jumps up on impact but returns to the steady state relatively quickly. In contrast, output jumps down on impact but returns to the steady state rather gradually. Initially, the systematic response of monetary policy is driven by a positive surprise in inflation and short rate goes up. But over time, a negative output gap induced by a positive markup shock plays a more important role and short rate gradually declines. This dynamic response of short rate implies that the short end of the yield curve will be much more responsive to a positive markup shock on impact, resulting in the flattening of the yield curve.

The above identification of macro factors for the level and slope of the yield curve is comparable with findings in other papers using different methods. For example, Diebold et al. (2006) associate the level and slope of the yield curve with inflationary expectations and cyclical variation of output, respectively. Although their analysis does not use structural restrictions from the DSGE model, the conclusion is similar to that drawn from this paper at the flexible price mode. The analysis in this paper provides additional insight for these links by highlighting the role of the response of the policy rate to macro variables and nominal rigidity.

Using a log-linearized New Keynesian model, Bekaert et al. (2010) argue that the time varying inflation target drives the yield curve level, whereas monetary policy shocks dominate the variations of the slope and curvature of the yield curve. Except for the interpretation of the yield curve slope, their conclusion is close to the analysis at the flexible price mode. Why is the interpretation of the yield curve slope different? Their estimates suggest that monetary policy is nearly insensitive to output gap. In contrast, I find that the coefficient of monetary policy with respect to output gap is small but positive. Once we shut down this channel, the deviation from the policy rule rather than the systematic response of the policy to real disturbances becomes a more dominant factor in explaining the fluctuations in the slope of the yield curve even in my model. In this case, real disturbances induce only very temporary movements in the short end of the yield curve without any meaningful change in the long end of the yield curve. Therefore, they cannot account for somewhat persistent variations in the slope of the yield curve that we observe from the data in Figure 5.

The difference in the estimate of the policy coefficient on output gap is due to the inclusion of pre-1980 data in Bekaert et. al. (2010). They show that the short-term interest rate did not increase in spite of a run-up in inflation and a persistent positive output gap during the 1970s. The finding suggests that policy responses were muted. It is not surprising that if we include data for that period, estimates of policy coefficients would become small.

Evans and Marshall (2007) conclude that the identified technology shock shifts expected inflation and the entire yield curve. This paper does not find any significant role for total factor productivity shocks. However, there are some issues in directly comparing their analysis with the empirical analysis in this paper. First, their measured shocks are obtained from different model, hence, the internal consistency of identification schemes that they draw on is not clear. While technology shock and shock to the marginal rate of substitution are obtained using first-order conditions in a calibrated DSGE model, monetary policy shock and fiscal policy shock are obtained from structural vector autoregressions in other papers. Quantitative implications of these shocks will be different if all the shocks are obtained from using restrictions of a single DSGE model. Second, persistent changes in monetary policy, that are often emphasized to explain the yield curve movement as in Kozicki and Tinsley (2005), are excluded in their model. Since they do not consider any persistent shock to the inflation target of the central bank, it forces the model to explain inflation persistence mainly by real shocks.

### 5.3 Time variation of term premium

Equilibrium bond yields in the nonlinear model contain a channel that can generate the time variation of term premium. To determine the significance of this channel, I compute the model implied measure of the term premium of the five-year bond yield by the deviation of the five-year bond yield from the average short rate expected over the five-year horizon,

$$TP_{n,t} = y_{n,t} - E_t\left(\frac{\sum_{j=0}^{n-1} i_{t+j}}{n}\right). \tag{25}$$

Model implied expectations are computed by averaging 1,000 simulated paths of future state variables at each time point. For comparison, I compute the term premium implied by a first-order vector autoregression (VAR (1)). I run a VAR (1) for output, inflation, and bond yields and compute expected future short rates based on the estimated coefficients in the VAR (1). The correlation between the two measures is weak (0.1239) at the flexible price mode. The sample mean term premium is much smaller in the DSGE model at both local modes, compared to the value obtained in the VAR (1). In addition, the volatility of the term premium measure implied by the DSGE model is roughly 20 basis points at the flexible price mode, which is much smaller than 121 basis points implied by the VAR (1) model. Doh (2006) shows that without generating a counter-factually high volatility of inflation, a standard DSGE model cannot generate the variation of term premium comparable to the evidence documented in reduced-form models. Rudebusch and Swanson (2008) explore DSGE models with various frictions and reach a similar conclusion.[21]

# 6 Conclusion

This paper estimates a small-scale New Keynesian model to identify macroeconomic sources of the yield curve. Unlike empirical factor models of the yield curve, this paper assumes that macro shocks in the DSGE model can explain the joint behavior of macro and term structure variables. I solve the macro model with a second-order approximation to equilibrium conditions and propose new closed-form solutions of bond prices given the second-order approximation to the macro model.

This paper finds that the estimated degree of nominal rigidity is important for identifying macro factors driving yield curve fluctuations. When the estimated nominal rigidity is low, the inflation target of the central bank drives persistent movements of inflation and shifts the entire yield curve, while markup shocks affect mainly shorter-term maturities. Accordingly, the level of the yield curve is determined by the time-varying inflation target of the central bank and the slope of the yield curve is driven by markup shocks. In contrast, markup shocks become highly persistent and drive low-frequency movements of inflation and bond yields when the estimated nominal rigidity is high.

The posterior distribution of parameters is bi-modal in terms of the degree of nominal rigidity. While the posterior mass of the sticky price mode is higher, survey data on inflation expectations seem to be more consistent with the flexible price mode. In fact, when I use survey data measures of inflation expectations in the estimation of the linear model, the posterior probability of the flexible price mode becomes much higher. Hence, a broad set of data support

[21] Amisano and Tristani (2009) show that incorporating recursive preferences into the DSGE model helps in amplifying the variation of the term premium in DSGE models. Nonetheless, they point out that this feature does not improve the time series fit for bond yields much compared to the DSGE model with little variations in the term premium. On the contrary, reduced form models tend to generate not only substantial variations of the term premium but also better fit for bond yields.

the flexible mode more than the sticky price mode. To a lesser degree than the inclusion of survey data information in the estimation, ignoring nonlinearities of the model solution also reduces the posterior probability of the flexible price mode. These findings indicate that incorporating additional information from observed proxies for inflation expectations and the nonlinearities of the model solution can be important for identifying macro shocks driving yield curve fluctuations.

## References


Amisano, G., Tristani, O. (2007): "Euro Area Inflation Persistence in an Estimated Nonlinear DSGE Model "*Manuscript*, European Central Bank.

Amisano, G., Tristani, O. (2009): "A DSGE Model of the Term Structure with Regime Shifts"*Manuscript*, European Central Bank.

An, S. (2005): "Bayesian Estimation of DSGE Models:Lessons from Second-order Approximations,"*Manusript*, University of Pennsylvania.

An, S., Schorfheide, F. (2007): "Bayesian Analysis of DSGE Models,"*Econometrics Reviews*, **26**(2-4), 113-172.

Ang, A., Bovin, J., Dong, S. (2007): "Monetary Policy Shifts and the Term Structure,"*Manusript*, Columbia University.

Ang, A., Piazzesi, M. (2003): "A No-arbitrage Vector-autoregression of Term Structure Dynamics with Macroeconomic and Latent Variables,"*Journal of Monetary Economics*, **50**, 745-787.

Bekaert, G., Cho, S., Moreno, A. (2010): "New-Keynesian Macroeconomics and the Term Structure,"*Journal of Money, Credit, and Banking*, **42**)(1), 33-62.

Bikbov, R., Chernov, M. (2010): "No-arbitrage Macroeconomic Determinants of the Yield Curve,"*Journal of Econometrics*, Forthcoming.

Binsbergen, J.H., Fernàndez-Villaverde, J., Koijen, R.S.J., Rubio-Ramírez, J. (2010): "Likelihood Estimation of DSGE Models with Epstein-Zin Preferences "*Working Paper*, National Bureau of Economic Research.

Canova, F., Sala, L. (2009): "Back to Square One: Identification Issues in DSGE Models "*Journal of Monetary Economics*, **56**(4), 431-449.

Chang, Y., Doh, T., Schorfheide, F. (2007): "Non-stationary Hours in a DSGE Model," *Journal of Money, Credit, and Banking*, **39**(6), 1357-1373.

Cooley, T. F., Prescott, E.C. (1995): "Economic Growth and Business Cycles,"in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton,NJ: Princeton University Press, 1-38.

Diebold, F.X., Rudebusch, G.D., Aruoba, B. (2006): "The Macroeconomy and the Yield Curve:A Dynamic Latent Factor Approach," *Journal of Econometrics*, **131**(1-2), 309-338.

Doh, T. (2006): "What Moves the Yield Curve? Lessons from an Estimated Nonlinear Macro Model" *Manuscript*, University of Pennsylvania.

Evans, C.L., Marshall, D. (2007): "Economic Determinants of the Nominal Treasury Yield Curve," *Journal of Monetary Economics*, **54**(7), 1986-2003.

Fernàndez-Villaverde, J., Rubio-Ramírez, J. (2007): "Estimating Macroeconomic Models:A Likelihood Approach," *Review of Economic Studies*, **74**(4), 1059-1087.

Fernàndez-Villaverde, J., Rubio-Ramírez, J., Santos, M. (2006): "Convergence Properties of the Likelihood of Computed Dynamic Models," *Econometrica*, **74**(1), 93-119.

Hördahl, P., Tristani, O., Vestin, D. (2008): "The Yield Curve and Macroeconomic Dynamics," *Economic Journal*, **118**(533), 1937-1970.

Jermann, U. (1998): "Asset Pricing in Production Economy," *Journal of Monetary Economics*, **41**(2), 257-275.

Johnson, N.L., Kotz, S. (1970): *Distributions in Statistics: Continuous Univariate Distributions -2*, Boston,MA: Houghton and Mifflin Company.

Kim, J., Kim, S.H., Schaumburg, E., Sims, C.A. (2008): "Calculating and Using second-order Accurate Solutions of Discrete Time Dynamic Equilibrium Models," *Journal of Economic Dynamics and Control*, **32**(11), 3397-3414.

Klein, P. (2005): "Second-order Approximation of Dynamic Models without the Use of Tensors," *Manuscript*, University of Western Ontario.

Kozicki, S., Tinsley, P.A. (2005): "Permanent and Transitory Policy Shocks in an Empirical Macro Model with Asymmetric Information," *Journal of conomic Dynamics and Control*, **29**(11), 1985-2015.

Koop, G., Pesaran, M.H., Potter, S.M. (1996): "Impulse Response Analysis in Nonlinear Multivariate Models," *Journal of Econometrics*, **74**(1), 119-147.

Litterman, R., Scheinkman, J.A. (1991): "Common Factors Affecting Bond Returns," *Journal of Fixed Income*, **1**, 51-61.

Lubik, T., Schorfheide, F. (2005): "A Bayesian Look at New Open Economy Macroeconomics" *NBER Macroeconomics Annual*, edited by M. Gerler and K. Rogoff, Cambridge,MA: MIT Press.

Ravenna, F., Seppälä, J. (2006): "Monetary Policy and Rejections of Expectations Hypothesis," *Manuscript*, University of California Santa Cruz.

Rudebusch, G.D., Swanson, E.T. (2008): "Examining the Bond Premium Puzzle with a DSGE Model ," *Journal of Monetary Economics*, **55**(s), 111-126.

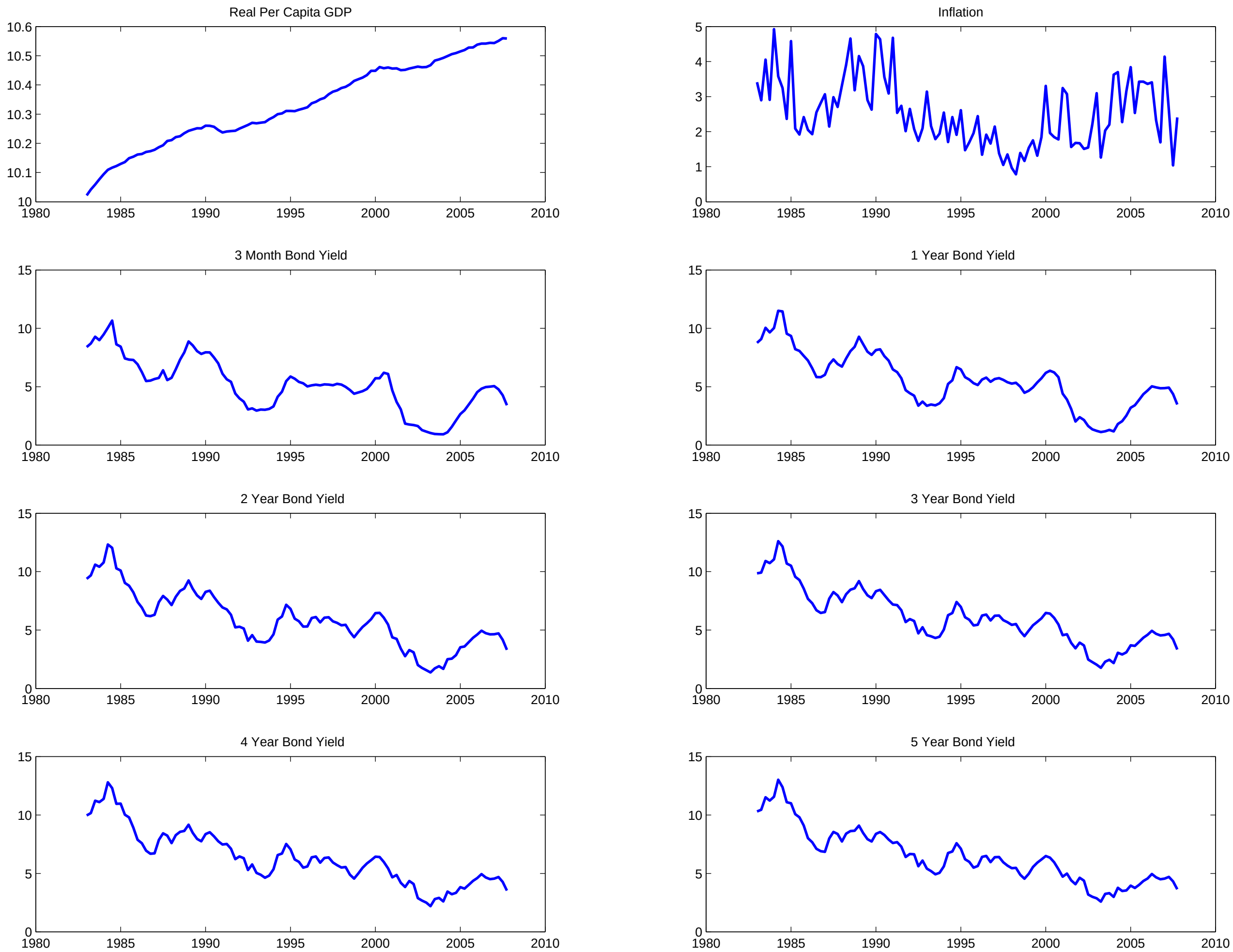
Rudebusch, G.D., Wu, T. (2008): "A Macro-Finance Model of the Term Structure, Monetary Policy, and the Economy," *Economic Journal*, **118**(530), 906-926.

Schmitt-Grohé,S., Uribe, M. (2004): "Solving Dynamic General Equilibrium Models Using an Second-order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, **28**(4), 755-775.

Singleton, K. (2006): *Empirical Dynamic Asset Pricing*, Princeton,NJ: Princeton University Press.

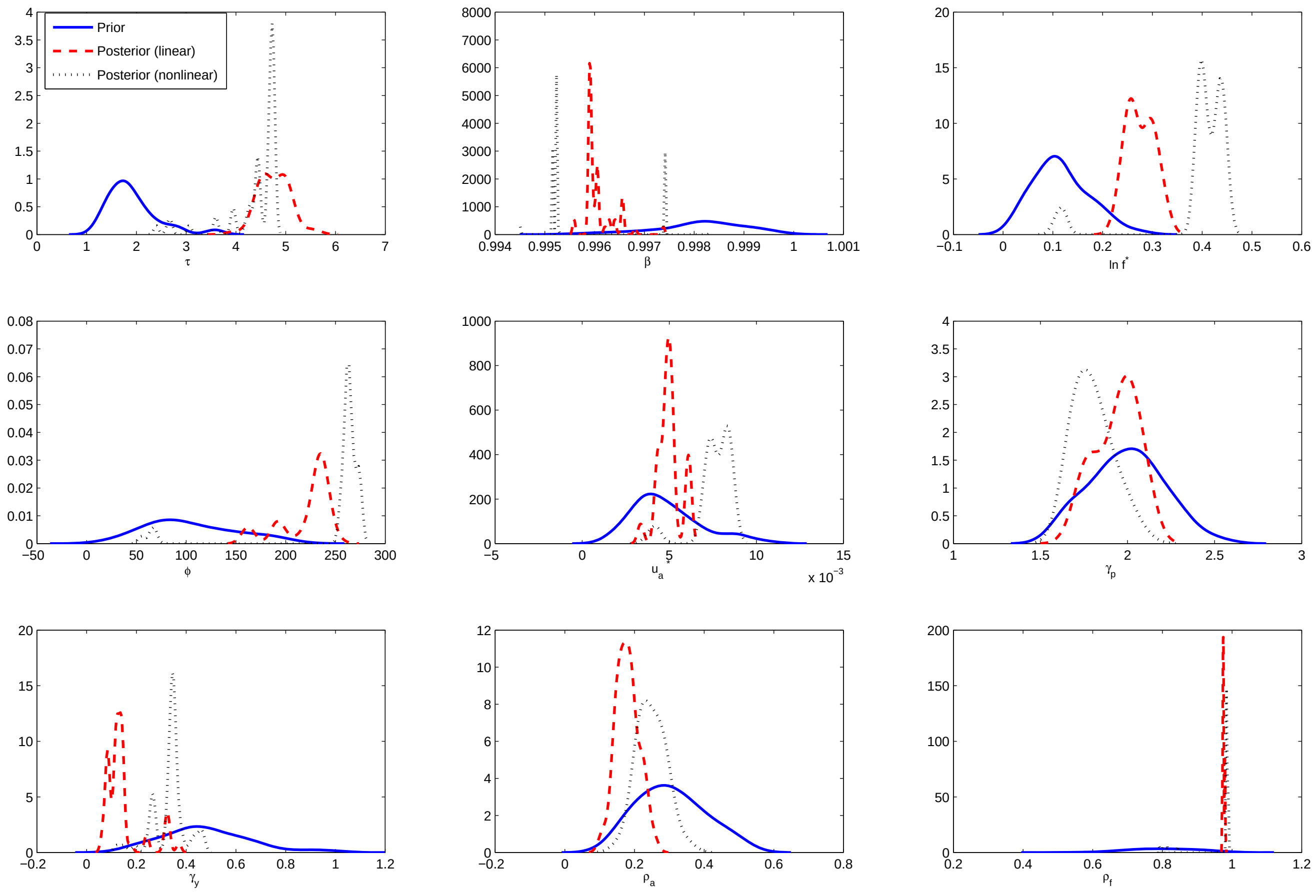
Woodford, M. (2003): *Interests and Prices: Foundations of a Theory of Monetary Policy*, Princeton,NJ: Princeton University Press.

Figure 1: Time Series Plots of Data



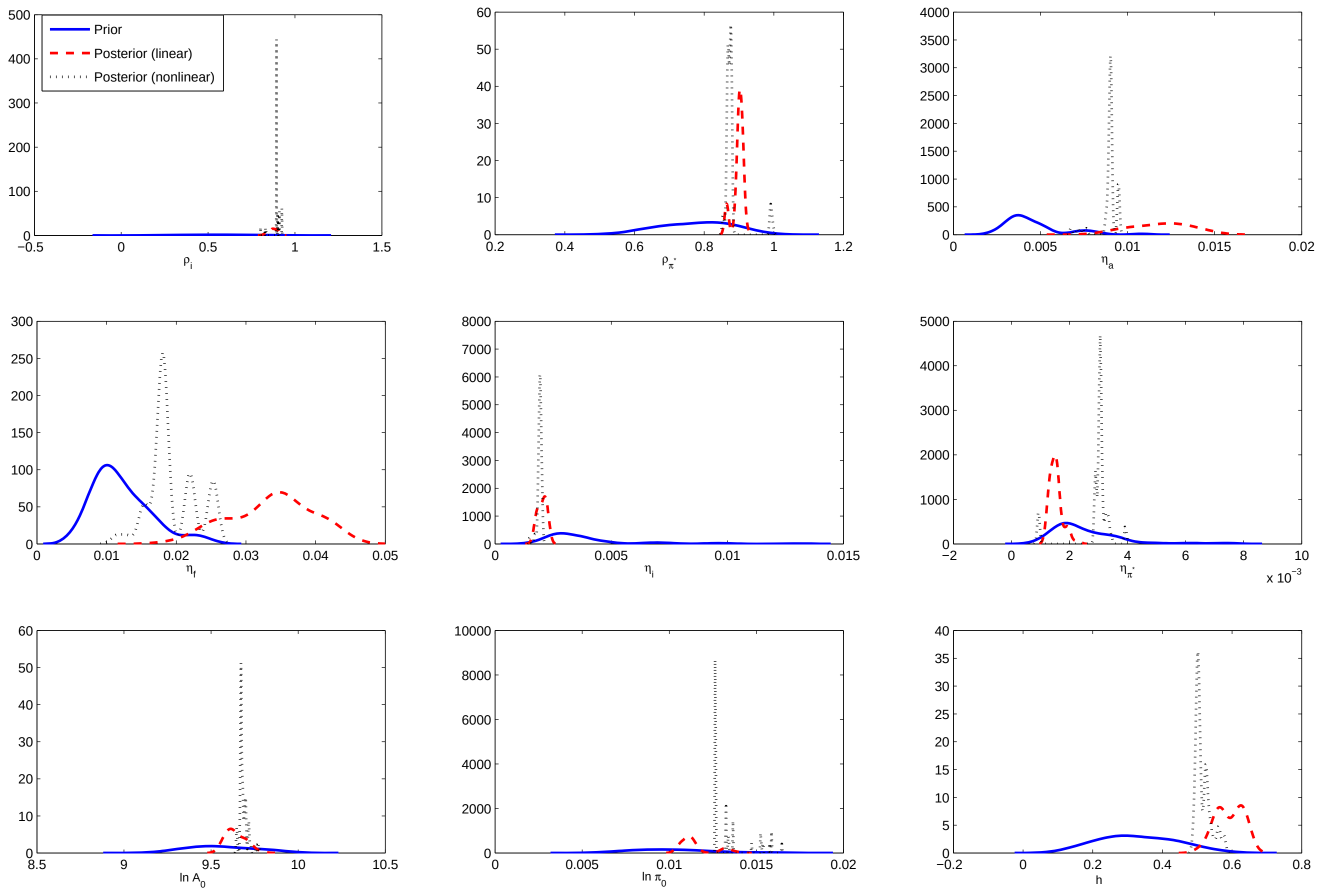

*Notes:* Inflation and bond yields are expressed in the annualized percentage rate.

Figure 2: Prior and Posterior Densities of Parameters I



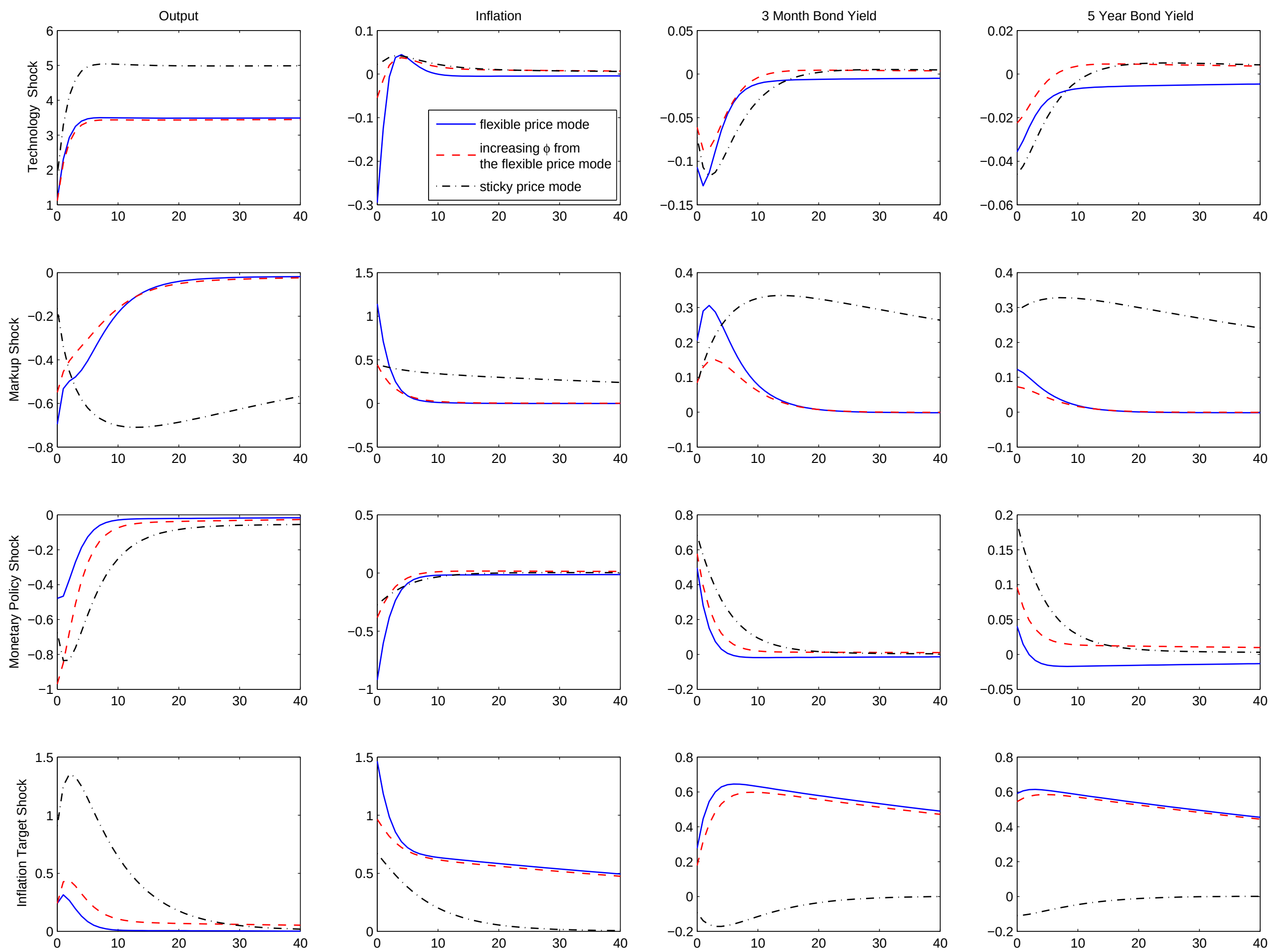

*Notes:* Kernel density estimates of parameters are based on 50 random draws from prior distributions and 50 posterior draws from an independent, mixed MCMC chain. Blue solid lines describe the output from prior draws, red dashed lines denote the output from the linear model, and black dotted lines denote the output from the nonlinear model.

Figure 3: Prior and Posterior Densities of Parameters II



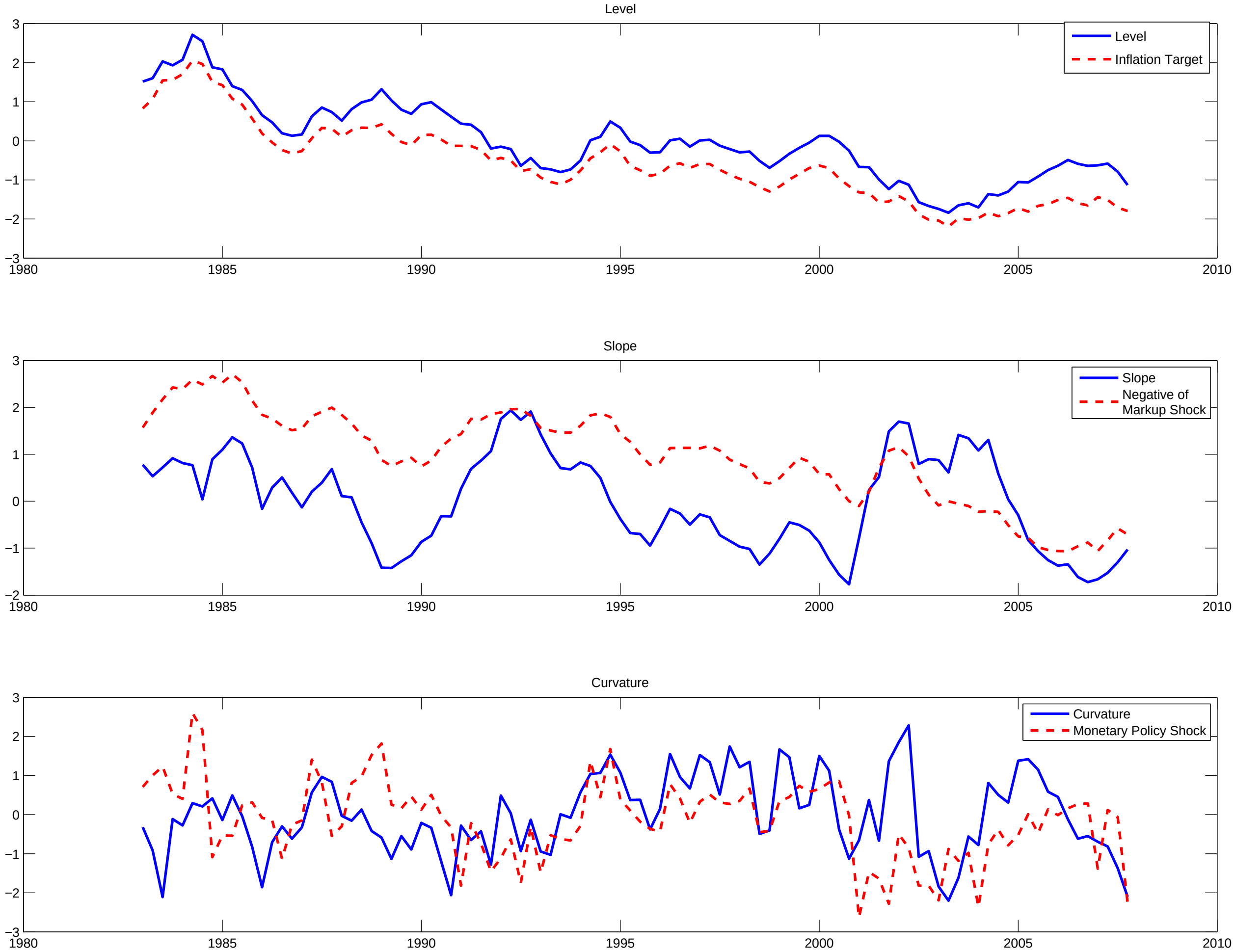

*Notes:* Kernel density estimates of parameters are based on 50 random draws from prior distributions and 50 posterior draws from an independent, mixed MCMC chain. Blue solid lines describe the output from prior draws, red dashed lines denote the output from the linear model, and black dotted lines denote the output from the nonlinear model.

Figure 4: Dynamic Impulse Responses




*Notes:* Mean values of generalized impulse responses are plotted. The blue solid lines describe results from the flexible price mode of the nonlinear model, the black dash-dot lines from the sticky price mode of the nonlinear model, and the red dashed lines from increasing only $\phi$ to the value in the sticky price mode with other parameter values are the same as in the flexible price mode.

Figure 5: Smoothed Estimates of Macro Factors at the Flexible Price Mode and Statistical Term Structure Factors




*Notes:*Smoothed estimates of macro factors are obtained at the flexible price mode of the nonlinear model.

Table 1: Prior Distributions

| Parameters | Domain | Density | Para(1) | Para(2) |
|---|---|---|---|---|
| $\tau$ | $\mathbb{R}^+$ | Gamma | 2 | 0.5 |
| $\beta$ | $[0, 1)$ | Beta | 0.998 | 0.001 |
| $\ln f^\star$ | $\mathbb{R}^+$ | Gamma | 0.11 | 0.05 |
| $\phi$ | $\mathbb{R}^+$ | Gamma | 100 | 40 |
| $u_a^\star$ | $\mathbb{R}^+$ | Gamma | 0.005 | 0.002 |
| $\gamma_p$ | $\mathbb{R}^+$ | Gamma | 2 | 0.2 |
| $\gamma_y$ | $\mathbb{R}^+$ | Gamma | 0.5 | 0.2 |
| $\rho_a$ | [0,1) | Beta | 0.3 | 0.1 |
| $\rho_f$ | [0,1) | Beta | 0.8 | 0.1 |
| $\rho_i$ | [0,1) | Beta | 0.5 | 0.2 |
| $\rho_{\pi^\star}$ | [0,1) | Beta | 0.8 | 0.1 |
| $\eta_a$ | $\mathbb{R}^+$ | Inverse Gamma | 0.004 | 4 |
| $\eta_f$ | $\mathbb{R}^+$ | Inverse Gamma | 0.010 | 4 |
| $\eta_i$ | $\mathbb{R}^+$ | Inverse Gamma | 0.003 | 4 |
| $\eta_{\pi^\star}$ | $\mathbb{R}^+$ | Inverse Gamma | 0.002 | 4 |
| $\ln A_0$ | $\mathbb{R}$ | Normal | 9.951 | 0.2 |
| $\ln \pi_0^\star$ | $\mathbb{R}^+$ | Gamma | 0.01 | 0.002 |
| $h$ | [0,1) | Beta | 0.3 | 0.1 |

*Notes:* Para (1) and Para (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; $s$ and $\nu$ for the Inverse Gamma distribution, where $P_{\mathcal{IG}}(\sigma|\nu, s) \propto \sigma^{-\nu-1} e^{-\nu s^2/2\sigma^2}$.

Table 2: LOCAL MODES OF THE POSTERIOR DENSITY

| Parameter | Flexible price mode | Sticky price mode |
|---|---|---|
| $\tau$ | 2.51 | 4.13 |
| $\beta$ | 0.997 | 0.995 |
| $\ln f^\star$ | 0.132 | 0.426 |
| $\phi$ | 72.9 | 271.6 |
| $400u_a^\star$ | 1.56 | 3.16 |
| $\gamma_p$ | 1.99 | 1.90 |
| $\gamma_y$ | 0.22 | 0.42 |
| $\rho_a$ | 0.205 | 0.290 |
| $\rho_f$ | 0.808 | 0.989 |
| $\rho_i$ | 0.839 | 0.912 |
| $\rho_{\pi^\star}$ | 0.992 | 0.866 |
| $400\eta_a$ | 2.76 | 3.56 |
| $100\eta_f$ | 1.55 | 1.54 |
| $400\eta_i$ | 0.68 | 0.76 |
| $400\eta_{\pi^\star}$ | 0.36 | 1.36 |
| $\ln A_0$ | 9.76 | 9.70 |
| $400\ln \pi^\star$ | 6.2 | 5.92 |
| $h$ | 0.54 | 0.52 |
| log posterior kernel | 4,085.7 | 4,095.2 |
| (log likelihood) | (4,078.6) | (4,091.3) |

Table 3: STANDARD DEVIATIONS OF NOMINAL VARIABLES: DATA VERSUS MODEL

| | $\pi_t$ | $y_{1,t}$ | $y_{4,t}$ | $y_{8,t}$ | $y_{12,t}$ | $y_{16,t}$ | $y_{20,t}$ |
|---|---|---|---|---|---|---|---|
| Data | 0.98 | 2.27 | 2.38 | 2.41 | 2.38 | 2.36 | 2.34 |
| Flexible Price Mode | 0.99 | 2.37 | 2.31 | 2.23 | 2.29 | 2.32 | 2.34 |
| Sticky Price Mode | 0.96 | 2.32 | 2.38 | 2.40 | 2.36 | 2.30 | 2.23 |

*Notes:* Standard deviations are in terms of the annualized percentage. Both modes are from the estimation of the nonlinear model. Standard deviations of variables from the model are computed based on the smoothed estimates of latent shocks.

Table 4: Correlations between Survey Expected Inflation and the Model Implied Expected Inflation

| | Correlation |
|---|---|
| Flexible Price Mode | 0.9163 |
| Sticky Price Mode | 0.7790 |

*Notes:* Survey expected inflation is the mean of GDP deflator inflation forecasts over the four quarters following the current quarter from the survey of the professional forecasters provided by the federal reserve bank of Philadelphia. The period is from 1983:Q1 to 2007:Q4. 1,000 simulations are used to compute the model implied one year ahead expected inflation.

Table 5: Log-likelihood of Local Modes: Linear Model with Survey Data

| | |
|---|---|
| Sticky Price Mode | 4,487.5 |
| Flexible Price Mode | 4,580.7 |

*Notes:* The linear model is estimated with the augmented dataset consisting of real GDP, inflation, short rate, five bond yields, and inflation expectations from survey data.

Table 6: Smoothed Estimates of Measurement Errors at the Flexible Price Mode

| | $\pi_t$ | $y_{1,t}$ | $y_{4,t}$ | $y_{8,t}$ | $y_{12,t}$ | $y_{16,t}$ | $y_{20,t}$ |
|---|---|---|---|---|---|---|---|
| mean absolute values (low $\phi$ ) | 0.033 | 0.180 | 0.192 | 0.183 | 0.128 | 0.122 | 0.151 |
| relative standard deviation | 0.047 | 0.099 | 0.103 | 0.101 | 0.073 | 0.066 | 0.082 |

*Notes:* Mean absolute values are in terms of the annualized percentage rate. Relative standard deviation is the standard deviation of measurement error of each variable divided by the standard deviation of that variable.

Table 7: Forecast Error Variance Decomposition

| variable | horizon | tech | markup | policy | target |
|---|---|---|---|---|---|
| | | Flexible Price Mode | | | |
| $\pi_t$ | 1 | 0.006 | 0.254 | 0.134 | 0.606 |
| $y_{20,t}$ | 1 | 0.004 | 0.051 | 0.002 | 0.942 |
| $\pi_t$ | 4 | 0.000 | 0.043 | 0.023 | 0.934 |
| $y_{20,t}$ | 4 | 0.002 | 0.024 | 0.000 | 0.974 |
| $\pi_t$ | 20 | 0.002 | 0.002 | 0.000 | 0.996 |
| $y_{20,t}$ | 20 | 0.002 | 0.002 | 0.000 | 0.996 |
| | | Sticky Price Mode | | | |
| $\pi_t$ | 1 | 0.000 | 0.380 | 0.065 | 0.555 |
| $y_{20,t}$ | 1 | 0.015 | 0.647 | 0.195 | 0.143 |
| $\pi_t$ | 4 | 0.003 | 0.486 | 0.032 | 0.479 |
| $y_{20,t}$ | 4 | 0.008 | 0.813 | 0.069 | 0.110 |
| $\pi_t$ | 20 | 0.000 | 0.979 | 0.000 | 0.021 |
| $y_{20,t}$ | 20 | 0.000 | 0.990 | 0.002 | 0.008 |

*Notes:* Posterior means of generalized impulse responses are used to compute forecast error variances. Forecast horizon is in terms of quarters.

Table 8: Regressions of Principal Components of the Yield Curve

| Regressors | Dependent Variables | | |
|---|---|---|---|
| | level | slope | curvature |
| Panel A: Regressions on observed macro variables | | | |
| constant | -12.180 (0.454) | 0.136 (0.184) | 0.031 (0.024) |
| $(\ln Y_t - \ln Y_{t-1})$ | 0.149 (0.060) | -0.099 (0.043) | 0.013 (0.008) |
| $\ln(1+\pi_t)$ | 0.267 (0.156) | 0.098 (0.063) | -0.013 (0.008) |
| $\ln(1+i_t)$ | 4.756 (0.161) | -0.239 (0.065) | -0.006 (0.009) |
| Adjusted $R^2$ | 0.927 | 0.113 | 0.038 |
| Panel B: Regressions on estimated macro shocks (flexible price mode) | | | |
| constant | 4.650 (0.131) | -0.819 (0.106) | -0.062 (0.021) |
| $\hat{u}_{a,t}$ | -0.284 (0.048) | 0.127 (0.039) | 0.003 (0.008) |
| $\widehat{\ln f_t}$ | 1.194 (0.082) | -0.610 (0.067) | -0.037 (0.013) |
| $\epsilon_{i,t}$ | 0.540 (0.059) | -0.262 (0.048) | 0.040 (0.010) |
| $\widehat{\ln \pi_t^\star}$ | 5.851 (0.093) | -0.279 (0.076) | -0.054 (0.015) |
| Adjusted $R^2$ | 0.993 | 0.640 | 0.168 |

*Notes:* Dependent variables are three principal components of the yield curve. Numbers in parentheses are estimates of standard errors. Estimated shocks are normalized so that variances are equal to 1.